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The Origin and Evolution of Large-Scale Structure in the Universe

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Contents

1. Introduction

Observational cosmology today is concerned with several questions. Perhaps the most exciting developments are the theoretical attempts to link physics of the early

Universe to grand unification theories of the fundamental forces of nature. A particularly influential recent development of this kind has been Guth's (1981) proposal that the very early Universe experienced a phase of exponential expansion, picturesquely called "inflation" by its author. Guth's idea provides an explanation of why the Universe is so homogeneous on large scales despite the fact that different regions did not appear to have been in causal contact (the "horizon" problem). It also explains at the same time why the Universe is geometrically "flat". Finally, the "inflationary Universe" leads to a prediction that the spectrum of initial density perturbations in the Universe, generated ultimately by vacuum fluctuations, should have equal power on all scales—the "Harrison–Zeldovich spectrum" (Guth and Pi 1982). These developments have justifiably exited considerable optimism that the way is open to understanding the physical history of the Universe from the Planck time (10^{-44} s) to the indefinite future.

However, we should recall Landau's remark that "cosmologists are often wrong but never in doubt" and remember that this dictum applies mainly to theoreticians. Thus we recognize that ideas like the "inflationary Universe" which points the way to the solution of several fundamental problems are ultimately of little value unless they are subject to observational or experimental test. Therefore, I shall focus attention on a particularly interesting general problem which is susceptible to observational attack using a variety of techniques and approaches. This is the problem of the origin and evolution of large-scale structure in the Universe. In my view this general topic illustrates the connection between the microscopic and macroscopic worlds and the connection between theory and observation in ways that are both beautiful and unexpected.

The study of the early Universe suddenly gained respectability as a serious science in 1965 with the surprising discovery of the microwave background radiation by Penzias and Wilson (1965). It is true that considerable theoretical work on a hot, compressed phase of the Universe had already been done by a few intrepid pioneers —in particular by Gamow and his collaborators who had predicted the existence of the background radiation (Alpher and Herman 1948, 1949). However, the discovery of the 2.7 K background showed directly that the Universe had evolved from an early, hot phase and, as Gamow had inferred on theoretical grounds, it had been initially radiation dominated in the sense that most of the mass-energy lay in the radiation and not in the matter. (No other plausible source for such a high photon density and no plausible mechanism for thermalizing the radiation from such hypothetical sources have been suggested.) Also, the background gave a value for the Universal photon/baryon ratio which is required for primordial nucleosynthesis calculations. This discovery was made three years after Bohr died; it was certainly the most important discovery in cosmology in the second half of the century and one of the most important in the whole of science because at one stroke it transformed the study of the nature and evolution of the early Universe into an observational science.

Historians used to speak of the "Dark Ages"—the Early Medieval period which lasted from the fall of the Roman Empire until the ninth century. The period was characterized by a retreat from urban life and was a time of relative chaos in European affairs after the order imposed by the Romans. In a similar vein there is at present a long dark age in our understanding of how the Universe evolved. It lasts from roughly 1 million years after the "Big Bang" when the microwave background-radiation was emitted until 2 billion or so years later when the first quasars formed. Like the terrestrial "Dark Ages", this period was also one of chaos —although its origin is still a mystery. More importantly it was the period in which the chaos developed into the large-scale structure which is so evident today. This epoch in the development of the Universe is literally dark as far as we are concerned because the gas in the Universe was too cool to shine and, according to current wisdom, no stars or galaxies with their nuclear sources of energy had yet formed.

Partly as a result of the existence of the Dark Ages, there are serious problems in understanding how structure formed in the Universe at all. The problem is that the theory of infinitesimal perturbations, worked out by Jeans early in the century, and which is successful in explaining how stars form from the static interstellar medium, runs into difficulties in the expanding Universe; the perturbations at the time of recombination (redshift $z = 1000$) are predicted on the most naive application of the theory to be larger than is consistent with the limits on angular fluctuations in the microwave background-radiation.

Fortunately, the problem is susceptible to observational attack. I shall describe several lines of work which are aimed at tracing the evolution of clustering in the Universe as far back as possible and at searching for the first objects to appear. At the other extreme, studies of the microwave background and of the relative abundances of the primordial elements are providing information about the physical state of the Universe as far back as a few seconds after the Big Bang.

2. The distribution of galaxies and quasars

2.1. Galaxies

The existence of galaxies outside our own Milky Way system was only established in 1925 when Hubble discovered Cepheid variable stars in the Andromeda nebula. Four years later Hubble discovered his redshift–distance relation which, following the earlier theoretical work by first Friedmann and then de Sitter, was interpreted in terms of an expanding Universe. The existence of clusters of nebulae had already been noticed as early as 1903 by Max Wolf; it was therefore immediately clear as soon as extragalactic astronomy was born that the galaxies were clustered, at least on scales of 1 Mpc or so. Later, it became evident that clustering extended the way from loose groups of galaxies such as our own Local Group up to compact clusters of hundreds or thousands of bright galaxies such as the nearby Virgo, Coma Berenices and Perseus clusters. The existence of clustering on still larger scales was a controversial subject for many years. Abell produced his catalogue of 2500 clusters of galaxies in 1957 using the plates of the first Palomar Sky Survey. The Abell clusters have a typical redshift of 0.15 (distance 1000 Mpc) and are exceptionally rich in galaxies—the Virgo cluster would not qualify for inclusion. Abell (1958) analyzed the distribution of his clusters on the sky and concluded that there was a probability of only 1 part in 10^{60} that they were not clustered. Abell also identified several examples of putative "superclusters." Nevertheless, a more sophisticated analysis of the same data by Yu and Peebles (1969) reached the conclusion that there was no significant clustering of the clusters. De Vaucouleurs (1956) studied the distribution of nearby, bright galaxies on the sky and concluded that the Local Group lies on the outskirts of an elongated, flattened structure called the Local supercluster or the Virgo supercluster, roughly centered on the Virgo cluster. Astronomers were slow to accept the importance or even the existence of the Local supercluster, but in recent years evidence has accumulated that superclusters are basic structures in the Universe. A recent review of their properties has been given by Oort (1983), who has played a particularly influential role in persuading astronomers that superclusters are important. The exploration of the Universe in three dimensions has also been facilitated by the very large redshift surveys carried out in the last few years. (As a measure of the rapid progress in this area, we recall that in 1956 Humason, Mayall and Sandage assembled a catalogue of redshifts measured up to that time; it contains 580 entries. A catalogue which is maintained by J. Huchra at the Center for Astrophysics currently contains redshifts for 7500 galaxies.) It is now possible to construct three-dimensional diagrams of the spatial distribution of nearby galaxies as inferred from their redshifts and their positions on the sky. The Local supercluster appears clearly in such diagrams with a density of galaxies which is about twice the mean smoothed-out density and, in general, the picture is one of elongated filaments containing the large clusters and enclosing apparently empty volumes. A similar picture emerges from deeper redshift surveys which have been conducted over restricted areas of the sky. There are almost no galaxies over a substantial range in redshift: the large-scale distribution of galaxies is once more revealed as taking the form of elongated structures separated by immense "voids". Note that the redshift gives a reliable indication of the distance of a galaxy if the peculiar motion is small. Galaxies in large clusters have velocity dispersions of about 1000 km s^{-1}. This produces a distortion of the inferred distribution along the line of sight, the so-called "fingers of God" effect, which distorts the true galaxy distribution. As a corrolary, the fact that the voids remain in such diagrams shows directly that the peculiar motions in the general field of a supercluster are small. There is now a growing realization that large voids in which at least intrinsically bright galaxies are sparse, are a basic feature of the distribution of galaxies. A large void discovered recently in the constellation Bootes by Kirshner et al. (1981) is about 100 Mpc in diameter.

Several years ago, Peebles and Hauser (1974) generated a remarkable map of the distribution on the plane of the sky of fainter and more distant galaxies from the Shane–Wirtanen counts. The Shane–Wirtanen catalogue goes out to a redshift of about 0.2 or to a distance of 500 Mpc. About 1 million galaxies were counted manually to produce this map; it will be some time before redshifts have been measured for all of them in order to obtain a deep two-dimensional map of the galaxy distribution over the whole northern sky! Nevertheless, even on the two-dimensional map, one gets the impression that the large-scale galaxy distribution has a cellular character.

Clusters of galaxies have recently been detected out to a redshift of about $z = 1$ and a few individual radio galaxies have been identified with redshift up to $z = 2$. A single galaxy with $z = 3.15$ has been found close to a quasar with the same redshift. Nothing is known about the clustering of galaxies at distances beyond a few hundred Mpc. Moreover, there is no direct evidence as to how galaxy clustering evolves in cosmic time.

2.2. Quasars

A typical quasar is about a factor of 10 brighter intrinsically than the brightest galaxies observed at the current epoch; it is likely that galaxies were brighter in the past, but by what factor is unknown. Thus at present most of our knowledge of the most distant parts of the Universe back to the end of the Dark Ages comes from studies of quasars. The relatively nearby quasars all appear to be in the centers of galaxies, probably mostly in galaxies of the spiral type. Thus, as has long been suspected, quasars appear to be scaled-up versions of the "Seyfert nuclei" which locally are found in a few percent of spiral galaxies. The highest quasar redshift found so far is $z = 3.78$ for the radio object PKS 2000-330 (Peterson et al. 1982). There is some doubt about whether quasars exist with much higher redshifts. A barrier (the "edge of the Universe") was suspected to exist at a redshift of $z = 2$ several years ago and then suddenly after a number of years of stasis, objects were quickly found with redshifts of $z = 3.44$ and 3.53, respectively. Hazard and the present author have made an intensive study of the redshift distribution of quasars in a particular field and have found that the number of quasars per unit redshift range begins to fall off at $z = 2$. Extrapolating our numbers beyond $z = 3.5$, we find that at $z = 4$ one would only expect one object per 30 square degrees on the sky. Although this means that there should be 1000 quasars with redshifts beyond $z = 4$ in the whole easily accessible sky, they would be hard to find and could well have escaped detection in the surveys carried out so far. It is of course very important to know if quasars suddenly switched on at a redshift of around $z = 4$ because this epoch, whenever it was, was likely to have been the epoch of galaxy formation as well.

Schmidt and others (see Schmidt and Green 1982) have shown that the co-moving space density of quasars has declined rapidly since the epoch corresponding to a redshift of $z = 2$—roughly by a factor of 1000. Moreover, it appears that the decline in number to the present day has been chiefly manifested by the intrinsically bright objects. Similar cosmological evolution is shown by the extragalactic radio sources (galaxies and quasars). It is interesting that the evolution in the number of these objects has occurred on the same time scale as the expansion of the Universe and not on a much longer or shorter time scale.

Quasars are not observed to be clustered; however, they are such rare objects even at high redshifts that this observation gives no interesting limit on the clustering of the associated galaxies at early times.

2.3. Clustering at large redshifts and the Lyman-alpha clouds

Our main quantitative information concerning the clustering of galaxies comes from the two-point correlation function of galaxies. The correlation function is defined as follows: Given a galaxy G1 we investigate the probability $\mathrm{d}P$ that there is a second galaxy G2 in a volume $\mathrm{d}V$, distance r from G1. It is found empirically from studies of the distribution of nearby galaxies that

$$\mathrm{d}P = \mathrm{d}V\, N_0[1 + \xi(r)], \tag{1}$$

where N_0 is the mean number density of galaxies per unit volume and the function $\xi(r)$ represents the enhanced effects of clustering. (For randomly scattered galaxies we would have $\mathrm{d}P = N_0\,\mathrm{d}V$.) Peebles found that the function $\xi(r)$ is a power law of the form (see Peebles 1980)

$$\xi(r) = \left(\frac{r}{r_c}\right)^{-1.77}, \tag{2}$$

where

$$r_c = 10\left(\frac{50}{H_0}\right)\ \mathrm{Mpc}. \tag{3}$$

Since the two-point correlation function depends only on r by hypothesis, it gives no information on the shapes of galaxy clumpings. Moreover, many different kinds of distributions are consistent with such a simple one-parameter description. Nevertheless, the two-point correlation function serves as a crude measure of clustering in the Universe which we can attempt to study at earlier epochs.

A particular interesting approach to the study of clustering at earlier times and large distances is afforded by the sharp absorption lines observed in the spectra of quasars. These lines can be divided into two distinct types (Sargent et al. 1980). Those redshifts containing lines of heavy elements as well as the Lyman series lines of H are believed to originate in the interstellar gas in galaxies distributed along the line of sight to the quasar. These "heavy element" redshifts are not common, having a density $\mathrm{d}N(z)/\mathrm{d}z$ of about 1 per unit redshift range ($\Delta z = 1$) at a redshift of $z \sim 2$. Such a rate of interceptions is compatible with the known space density of galaxies and the inferred sizes of their gaseous halos. A second category of quasar absorption lines contains only the Lyman lines of hydrogen with no detectable lines of heavier elements. These redshifts are about 50 times more common than the "heavy element" redshifts and produce a "forest" of Lyman-alpha absorption lines which extends from the Lyman-alpha emission line (i.e. from the redshift of the quasar itself) down to shorter wavelengths. Unlike the "heavy element" redshifts, the lines of the Lyman-alpha forest exhibit a strong redshift dependence—greater than that expected for objects of fixed cross section and with a fixed co-moving density which move apart as the Universe expands. Moreover, the lines of the "forest" are found to be completely randomly distributed on smaller scales (from 30–30 000 km s^{-1} or from 0 to 0.05 Hubble radii) while the "heavy element" lines show considerable clustering on small scales (< 200 km s^{-1}). Primarily, because of their lack of heavy elements it is currently supposed that the Lyman-alpha forest lines are produced by primordial intergalactic clouds. They appear to be confined by the pressure of an external medium rather than by the self-gravity and can be used to obtain important limits on the physical state of the intergalactic medium at large redshifts (see section 4).

For cosmologically distributed intervening objects, a clumping in redshift can be directly related to clumping in space even at large redshifts where the objects responsible for the absorption lines cannot be observed. The Lyman-alpha forest lines are so plentiful that they can be used to place a very fine limit on the value of r_c in the correlation function $\xi(r)$. The best limit so far obtained is $\xi \leq 0.2$ Mpc at $\langle z \rangle = 2.44$ (Sargent et al. 1982). It is not known how galaxy clustering evolves in time; however, on the hierarchial clustering model developed by Peebles and his associates (Peebles 1980), the correlation function retains the form $\xi(r) \sim r^{-1.77}$ provided the clustering is non-linear ($\delta\rho/\rho > 1$). If this is the case it is easy to show that r_c changes with redshift according to the relation

$$r_c(z) = r_c(z=0)(1+z)^{-5/3}. \tag{4}$$

Thus, on this simple hypothesis, it is expected that r_c should be 1.07 Mpc at $z = 2.44$. Therefore, the observed clustering of the Lyman alpha clouds at $z = 2.44$ is much weaker than expected. There are several possible explanations of this observation. One is that the Lyman-alpha clouds are confined to the voids in the galaxy distribution and so cluster much less strongly than galaxies at all epochs. Another is that the galaxy clustering developed late and that the Lyman-alpha clouds share the distribution of galaxies. Future observations from the Space Telescope of Lyman-alpha clouds at low redshifts will solve the problem of their distribution. In the meantime it is noteworthy that the most distant objects that we can study show no signs of clustering.

3. *The cosmological density parameter* Ω

3.1. Definitions

In addition to observations of the galaxy distribution, information on the overall distribution of mass in the Universe can be obtained from studies of dynamics of aggregates of galaxies on all scales on which they are clustered. The Friedmann equations lead to the notion of a critical mean smoothed out density in the Universe

$$\rho_c = \frac{3H_0^2}{8\pi G}, \tag{5}$$

where H_0 is the present value of the Hubble expansion constant. The value of H_0 is about 50 km s^{-1}/Mpc (Sandage and Tammann 1984); its value is in dispute in the range $40 < H_0 < 90$ km s^{-1}/Mpc (Rowan-Robinson 1985). It is usual to define $\Omega_0 = \rho_0/\rho_c$ where ρ_0 is the actual density of the Universe at the present epoch. As is well known, if $\Omega_0 < 1$ the Universe is open and will expand forever, if $\Omega_0 > 1$ it will eventually reverse its expansion and experience the "Big Crunch." A Universe with

$\Omega_0 = 1$ is geometrically flat and is the value which appears as the natural outcome of the "inflationary" scenario (Guth 1981) and its subsequent modifications (Albrecht and Steinhardt 1982) which attractively explain the Universe's present day isotropy and its close approach to flatness.

In general, Ω changes with time as the Universe expands. In a model with zero cosmological constant

$$\Omega(z) = \frac{\Omega_0(1+z)}{1+\Omega_0(z)}. \tag{6}$$

Thus for $z \gg 1$

$$\Omega - 1 = \frac{1}{z}\left(1 - \frac{1}{\Omega_0}\right). \tag{7}$$

As many authors have remarked, it is strange that Ω_0 is so close to unity at the present epoch; it must have been very close to unity in the past.

Current estimates of Ω_0 lead to the result that the Universe contains a preponderance of "dark matter"; considerations which we shall outline later indicate that this must be primarily in the form of non-baryonic matter.

3.2. *Mass-to-light ratios*

A summary of our present knowledge of Ω_0 is as follows. Astronomers find it convenient to discuss masses and mass densities in terms of the mass-to-light ratio, M/L, in solar units. Dynamical studies of the central parts (say the inner 10 kpc) of galaxies, both spiral and elliptical, lead to values $M/L = 6$. This appears as reasonable, because stars similar in type to the sun (with $M/L \sim 1$) appear to dominate the spectrum of most galaxies.

In the solar neighborhood studies of the motions of stars lead to $M/L = 3$, although only about 50 percent of the mass can be accounted for in visible stars (Oort 1960).

It has been shown in the last few years that almost invariably spiral galaxies exhibit flat rotation curves in which the rotational velocity $V_r \sim r$ out to as far as it can be observed (approaching 100 kpc radius in some cases). Such a flat rotation curve leads to an inferred mass distribution in which the mass inside radius r, $M(<r) \sim r$, i.e., the total mass diverges logarithmically. On the other hand, the projected luminosities of spiral galaxies are observed to fall off exponentially,

$$L(r) \sim e^{-\alpha r}, \tag{8}$$

with a scale length $1/\alpha$ which is typically a few kpc. Thus, the inferred M/L ratios of spiral galaxies rise with increasing radius and approach values $M/L \sim 100$ in their extreme outer parts (100 kpc scale). Accordingly, spiral galaxies are now considered to be luminous condensations in extended "dark halos" of non-luminous matter.

The dynamical studies of the motions of galaxies in small groups of a few galaxies and in large clusters of hundreds of objects also lead to mass-to-light ratios in the range $M/L = 100$ to 300. This is on scales of a few hundred kpc to a few Mpc.

Recently studies have been made of the local anisotropy of the local "Hubble-flow" introduced by our relative proximity (15 Mpc) to the Virgo cluster of galaxies: this also leads to a value of $M/L \sim 300$ on scales of tens of Mpc.

3.3. Dark matter

The mean luminosity density in the local vicinity is $L = 8 \times 10^7\ L_\odot\ \mathrm{Mpc}^{-3}$. This leads to a relationship between Ω_0 and M/L of the form

$$\frac{M}{L} = 840\ \Omega_0 \frac{H_0}{50}. \tag{9}$$

Accordingly, if $M/L = 300$ locally, then $\Omega_0 \sim 0.3$. However, we have seen that the M/L ratios of the luminous parts of galaxies are small, $M/L \sim 6$. Thus the visible matter in the Universe has $\Omega_0 \sim 0.01$ and therefore contributes only a small fraction of the total mass density. Whatever its form, most of the mass in the Universe is dark. It will be observed from the preceding discussion that M/L and Ω_0 appear to increase with the scale on which they are measured from a few kpc to tens of Mpc.

In general, dynamical analyses cannot distinguish mass distributions which are smooth on a scale larger than the scale of the test particles being used for the mass determination. Accordingly, attempts are now under way to determine M/L and Ω_0 on scales as large as 100 Mpc. A powerful observation in this regard is that the direction indicated by the dipole anisotropy in the microwave background radiation is now well established to correspond to a velocity of $V = 600 \pm 50\ \mathrm{km\ s^{-1}}$ in a direction 45° away from the nearest large mass concentration, the Virgo cluster. Part of this velocity is due to our "infall" into the Virgo cluster at 200–300 $\mathrm{km\ s^{-1}}$: this leads to a value of $\Omega_0 = 0.2$. It now appears that the Local Group of galaxies and the entire Virgo cluster are in turn moving towards a more distant large concentration of galaxies in Hydra-Centaurus at 400 $\mathrm{km\ s^{-1}}$. If confirmed this would lead to a very large value of $\Omega_0 \sim 0.5$ on a scale of 100 Mpc. The details of these large scale motions are currently being investigated via mapping of the local velocity field through large redshift surveys.

3.4. Primordial nucleosynthesis

The discovery of the microwave background radiation made it possible to begin accurate calculations of conditions in the Universe; a particularly fruitful outcome has been detailed calculations of primordial nucleosynthesis yields as a function of the assumed baryon density. The isotopes D, He^3, He^4, and Li^7 are all synthesized in the first 1000 seconds of the expansion of the Universe. The resulting abundances of these isotopes are sensitive in varying degrees to the mass density in the form of baryonic matter Ω_b^0. For example, the higher values of Ω_b^0 result in more He^4 and less D. A recent critical review of the observed abundances led to values of Ω_b^0 in the range 0.1 to 0.14 (Steigman and Boesgaard 1986). There are considerable difficulties in measuring the abundances of all of these elements and in extrapolating back to the "primordial" values before nucleosynthesis by stars became important. Nevertheless, the present abundance measurements (all of which have difficulties) are consistent with the same value of $\Omega_b^0 \sim 0.1$.

3.5. Summary

Dynamical studies of the motions of galaxies on supercluster scales (~ 30 Mpc) indicate that the cosmological density parameter has a value $\Omega_0 \sim 0.3$ at the present epoch. Visible matter only contributes $\Omega_0 \sim 0.01$. The primordial nucleosynthesis studies show that the baryonic contribution to Ω_0 cannot exceed 0.15. Thus, if $\Omega_0 = 1$ as is demanded by the "inflationary" Universe scenario (Guth 1981), then it cannot be in the form of baryons. Moreover, if the dark matter is present in sufficient amount to close the Universe it must be distributed on scales larger than that of superclusters.

4. The intergalactic medium

The Lyman-alpha forest of absorption lines in the spectra of quasars has been interpreted as being due to intergalactic clouds of very low heavy-element content. We introduced these clouds in section 2.3 where we also discussed their clustering properties. The Lyman-alpha clouds appear to be of galactic dimensions; a beautiful observation of the similarity in Lyman-alpha absorption lines in the spectra of two images of the gravitationally lensed quasar Q2345 + 007 shows that they have a diameter of about 10 kpc (Foltz et al. 1984). Theoretical studies show that the clouds are not supported by their own self-gravitation, but instead must be confined by a general intergalactic medium whose properties can be estimated from the inferred physical state of the Lyman-alpha coulds themselves (Sargent et al. 1980, Ostriker and Ikeuchi 1983). In summary, the condition that the clouds are ionized by the metagalactic quasar flux and the requirement that they are not ablated on a short time scale by the surrounding hot medium leads to the conclusion that the clouds themselves have masses of about 10^8 $M_\odot$ (of the same order as dwarf galaxies) and are highly ionized with an ionization fraction for hydrogen $N(\text{H-II})/n(\text{H-I}) = 10^5$. The clouds have a temperature of $T_c = 3 \times 10^4$ K and a density of $n_c = 10^{-4}$ electrons and ions per cm^3. They only contribute a negligible fraction to the total mass density of the Universe—$\Omega_c \sim 10^{-3}$. The general intergalactic medium required to confine the clouds has a density $n_c^M = 10^{-5}$ cm^{-3} and a temperature of $T_M = 3 \times 10^5$ K at a redshift $z = 2.4$. Since the Universe is expanding, this general intergalactic gas must be cooling and becoming more tenuous with cosmic time. It can easily be estimated that at the current epoch $T_M^0 \sim 10^3$ K and $n_M^0 \sim 2 \times 10^{-7}$ cm^{-3}. The intergalactic gas contributes a larger fraction of the mass density of the Universe than the Lyman-alpha clouds, namely $\Omega^M \sim 0.1$ for $H_0 = 50$ km s^{-1}/Mpc

—comparable to the visible galaxies. (Note that the densities involved are so low that the intergalactic gas stays highly ionized as it cools because the recombination time is larger than the Hubble time.) At the low temperatures and densities quoted, it will be very difficult to make direct observations of the intergalactic gas. It is noteworthy, however, that if the inferred properties of the Lyman-alpha clouds are anywhere near the true situation, these objects could not co-exist with a hot, dense intergalactic medium ($T^{\mathrm{M}} \sim 10^8$ K; $\Omega^{\mathrm{M}} \sim 1$) which is required to explain the diffuse X-ray background.

The Lyman-alpha clouds thus enable us to establish that there cannot be a large baryonic contribution to the cosmological density parameter in the form of hot intergalactic gas. Also, the process of galaxy formation must have been relatively efficient, with ~ 50 percent of the initial gas being converted into galaxies.

5. The microwave background radiation

The discovery of the 2.7 K microwave background radiation has had an enormous impact on cosmology since it enables us to make direct observations of the state of the Universe at a redshift of $z = 1000$ and at a cosmic time only 10^5 years after the Big Bang.

The radiation peaks at a wavelength of about 1 mm; however, it is most easily studied from the ground at wavelengths of 1 cm and above, because of variable atmospheric transmission problems at shorter wavelengths. A wavelength of about 1 cm is ideal because the Galactic background radiation is falling as $\nu^{-0.7}$ while the microwave background radiation spectrum is rising as ν^2 in this wavelength region. Thus, particularly for studies of the angular fluctuations in the background, a wavelength of about 1 cm is optimal.

The scales of interest for studies of early galaxy formation are obtained from the following considerations. A scale of 1 min corresponds to $\sim 10^{11}\ M_{\odot}$, the size of a galaxy. A scale of 10 min corresponds to $\sim 10^{14}\ M_{\odot}$, about the size of a cluster of galaxies. Finally, 1° corresponds to $\sim 5 \times 10^{16}\ M_{\odot}$, the scale of a supercluster. A radio telescope with a diameter of 40 m has a beam with a FWHM size of 1 min at 1 cm. Thus, such a telescope is ideal for studies of angular fluctuations on scales of a few minutes of arc which correspond to the scales of clusters of galaxies.

The only anisotropies in the microwave background radiation which have been discovered so far are the "dipole" anisotropy due to the Solar System's peculiar motion in the Universe and the smaller scale anisotropy seen in the direction of clusters of galaxies due to the Sunyaev–Zel'dovich effect. (In this last effect the hot electrons in the intergalactic gas in a cluster of galaxies scatter the microwave background radiation to high frequencies, resulting in a diminution or "cooling" of the radiation in the Rayleigh–Jeans part of the spectrum.) The dipole anisotropy, $\Delta T/T \sim 2 \times 10^{-3}$, implies that the Local Group of galaxies is moving at a velocity of ± 600 km s^{-1} in a direction some 45° away from the Virgo cluster: the implications of this result for Ω were discussed in section 3.4.

On smaller scales, in directions away from clusters of galaxies, increasingly sensitive measurements have so far not succeeded in detecting any anisotropy. Such measurements present a difficult technical challenge. Even at 1 cm, variable absorption by atmospheric water vapor is a problem. Also the side lobes of the antenna see the surrounding ground which is a temperature of 300 K. Hence, the observations must be made at a cold, dry site and the antenna moved as little as possible to eliminate variable ground spill-over. A particular useful technique is to point an altazimuth mounted telescope at the N pole and to alternate it between two directions in the sky close to the pole by moving it only in azimuth. At the same time two feeds are Dicke-switched. Such measurements have been reported by Uson and Wilkinson (1984) and are currently being carried out by Readhead, Sargent, and Moffet (1986). In typical measurements, the Dicke switching is done between two points 5–7 min apart in azimuth, while an annulus 1.5 min wide is traced out by the rotation of the earth in a 24-hour period. The best limits are $\Delta T/T < 3.8 \times 10^{-5}$ on a scale of 7.1 min (corresponding to about 10^{14} $M_{\odot}$ at $z = 1000$.) It is hoped that eventually the limits obtained by this technique can be lowered by a factor of 10 on the ground. In the meantime, the present limits already pose serious problems as shall be seen in section 6.3.

6. *Theory of fluctuations*

6.1. *Adiabatic and isothermal fluctuations*

It is supposed that the present clumpiness in the distribution of galaxies is the result of the evolution of primordial fluctuations, since the Universe is known to have been in a gaseous form at the time when matter and radiation decoupled at $z = 1000$. For several years it has been hoped that a theory of galaxy formation would be devised in which infinitesimal primordial perturbations in the density of matter would give rise eventually to galaxies. In fact, this approach leads to conflicts with the observations.

Two types of primordial fluctuations have been considered. In "adiabatic fluctuations" both matter and radiation are clumped. In "isothermal" fluctuations only matter is clumped. Adiabatic fluctuations are thought to be more natural because it is expected that a fundamental theory of elementary particles would lead to the prediction of a specific ratio $n_{\text{photon}}/n_{\text{baryon}}$ which would be the same everywhere at some early stage of the Universe.

The first-order question in the theory of galaxy formation is: which came first—superclusters or galaxies? On the hierarchial clustering scenario which has been explored particularly by Peebles and his associates, galaxies, or even globular clusters, form first and are subsequently aggregated by gravity into clusters and superclusters. This kind of picture would result from isothermal initial perturbations.

The other extreme view is that supercluster-sized objects (called "pancakes" by Zel'dovich) fragmented to form galaxies. This scenario would result from adiabatic perturbations.

The spectrum of the initial fluctuations is important in determining the range of masses which are eventually produced. The Fourier spectrum of density fluctuations

$\delta = \delta\rho/\rho$ as a function of wave number κ is assumed to be of the power-law form

$$|\delta_\kappa|^2 = \kappa^n. \tag{10}$$

This translates into a mass spectrum

$$S = \kappa M^{-\alpha}, \tag{11}$$

where κ is a constant whose value is $\kappa < 10^{-4}$ (from the absences of a quadrupole to the anisotropy of the microwave background) and where $n = 6\alpha + 1$.

6.2. *Growth of fluctuations*

The value $\alpha = 0$, or $n = 1$ is known as the Harrison–Zel'dovich or "constant curvature" spectrum. This particular fluctuation spectrum is predicted to emerge from quantum vacuum fluctuations in the "inflationary" scenario. In a matter dominated Universe infinitesimal density fluctuations are found to grow with time relative to the smoothly expanding background as

$$\delta = \delta\rho/\rho \sim t^{2/3}, \tag{12}$$

exactly the same time dependence as the scale factor $R = R_0/(1+z)$. On the other hand, in a radiation dominated Universe it is found that

$$\delta \sim t. \tag{13}$$

In an adiabatic fluctuation

$$\delta_R = \tfrac{4}{3}\delta_B. \tag{14}$$

The first result of the discovery of the microwave background was to give a value for the ratio of photons to baryons,

$$n_P/n_B \sim 10^9, \tag{15}$$

a quantity which is conserved in the expansion of the Universe. (The number density of photons produced by stars, etc., is lower than the primordial number by about 5 orders of magnitude.)

It can be shown that a growing fluctuation continues to expand, although at slower rate than the background Universe until it has reached a maximum size which, for $\Omega = 1$, is 5.6 times its initial size. It then begins to collapse to a galaxy, cluster, or supercluster depending on the scale of the initial fluctuation. However, the analysis of which fluctuations can grow is more complicated in the case of the expanding Universe than in the case of the interstellar medium. The first point is that fluctuations can only grow when they are causal—that is when they are smaller than ct at time t after the Big Bang. This happens for a galaxy-sized fluctuation at about one year and a supercluster at about $t = 10^5$ years. Secondly, fluctuations can only grow if they are larger than the Jeans length

$$\lambda_J = \frac{2\pi}{\kappa} = \left(\frac{\pi \kappa T}{\mu G}\right)^{1/2}. \tag{16}$$

The corresponding Jeans mass is

$$M_J = \tfrac{4}{3}\rho\left(\frac{\lambda_J}{2}\right)^3, \tag{17}$$

During the radiation-dominated era, the cosmic fluid is relativistic so that

$$M_J \sim M_H. \tag{18}$$

Between the onset of the matter-dominated era and recombination M_J levels out at $\sim 10^{17}$ solar masses—which is larger than the mass of a supercluster. Before this period fluctuations cannot grow, but oscillate like acoustic waves. However, photon diffusion damps small adiabatic fluctuations (this is known as "Silk damping"). The critical damping length is at time t,

$$d_S \sim \left(\frac{ct}{n_e \sigma_T}\right)^{1/2}, \tag{19}$$

where n_e is the electron density and σ_T is the Thomson scattering cross-section. The corresponding "Silk mass", below which fluctuations are damped out is

$$M_S = \tfrac{4}{3}\pi\rho_m d_S^3, \tag{20}$$

where ρ_m is the baryon density or

$$M_S \sim 1.3 \times 10^{12} \, (\Omega h^2)^{-3/2} M_\odot. \tag{21}$$

Thus, for $\Omega_b = 0.1$, $h = 1/2$, $M_S = 3 \times 10^{14}\ M_\odot$.

As we have seen, between t_{eq} and the recombination time t_r, M_J levels out at about $10^{17}\ M_\odot$. Thus under the adiabatic picture only large initial structures can form, i.e. those with masses in the range 10^{14}–$10^{17}\ M_\odot$. At recombination the Jeans mass suddenly falls to $10^6\ M_\odot$; however, small fluctuations have already been damped out.

Thus, adiabatic fluctuations lead to large structures which form first. These are then supposed to fragment into galaxies. This attractive picture is the basis of Zel'dovich's "pancake" theory for the formation of galaxies via superclusters.

6.3. Problems with the simple adiabatic picture

As we have seen, fluctuations $\delta = \delta\rho/\rho$ grow linearly with the scale factor $R = R_0/1+z$ while the Universe is matter-dominated. However, only fluctuations with $M > M_J \sim 10^{17}\ M_\odot$ can grow until the time of recombination when the matter temperature suddenly falls. The subsequent growth of an initial density fluctuation δ slows when the Universe begins to expand freely: this happens at $z \sim \Omega^{-1}$. Accordingly, in a baryon dominated Universe with $\Omega_b = 0.1$, δ only grows from $z = 1000$ (the value at recombination) to $z = 10$, i.e. by a factor of 100.

Now the value of $\Delta T/T$, to be expected in the microwave background at recombination when the radiation experiences its last scattering, is related to the size of the corresponding density fluctuations by

$$\frac{\Delta T}{T} = \frac{1}{3}\frac{\Delta\rho}{\rho}. \qquad (22)$$

Thus we expect that, since $\Delta\rho/\rho \sim 1$ at the present epoch it must have been $\Delta\rho/\rho = 10^{-2}$ at recombination. The corresponding value of $\Delta T/T \sim 3 \times 10^{-3}$ should be observed on scales larger than the Silk mass $M_S \sim 10^{14}\ M_\odot$; this corresponds to an angle on the sky of a few arcminutes.

Observations of the microwave background show that fluctuations on this scale are less than 3×10^{-5}—a factor of 100 smaller than is expected on the simplest picture based on adiabatic perturbations.

There are several possible loopholes in the above argument. It is possible that isothermal fluctuations which suffer no Silk damping and which lead to the formation of globular cluster-sized ($M \sim 10^6\ M_\odot$) primordial objects were generated on scales much larger than the causal horizon at early times. A second possibility is that matter was re-ionized some time after recombination. If this occurred at $z > 10$ it would have washed out the fluctuations $\Delta T/T$ produced by proto-pancakes. However, no source of ionizing radiation is known earlier than $z = 3.8$ where the earliest quasars are observed.

The most likely hypothesis is that non-baryonic dark matter preponderates in the Universe. Such matter, if it interacts weakly with baryonic matter, could begin to clump early, before the baryons were released from the radiation field. The baryonic matter would then fall into the potential wells created by the dark matter. Such a scenario would avoid the conflict with observations of the microwave background which is seen with the simple adiabatic picture in a baryon-dominated Universe.

7. *Simulations of galaxy clustering*

7.1. *Types of dark matter*

We have seen that studies of the motions of galaxies on large scales have indicated that "dark" matter predominates the mass density of the Universe. Active research is under way at the present time to try and determine the nature of the dark matter. A particularly promising avenue of approach is the study of the evolution of galaxy clustering in the Universe under various hypotheses concerning the nature of the dark matter.

The dark matter candidates can be divided into three broad categories: "cold", "warm" and "hot".

Hot dark matter is the term given to abundant, light particles, weakly interacting with baryonic matter, which remain relativistic as the Universe expands until slightly before the era of recombination. A candidate for such a particle would be a massive neutrino with mass in the range $10\ \text{eV} < m_\nu < 100\ \text{eV}$. It can be shown that freely streaming relativistic particles erase density fluctuations on a scale smaller than the cosmological horizon: for massive neutrinos, fluctuations would have damped out if their present scale is less than the critical damping scale λ_c which, at the present epoch translates to

$$\lambda_c = 41\left(\frac{m_\nu}{30\ \text{eV}}\right)\ \text{Mpc}. \tag{23}$$

Warm dark matter consists of particles which interact more weakly than neutrinos, which are less abundant than neutrinos and which have a mass of around 1 keV. Candidates for such particles are the as yet hypothetical gravitinos and photinos. Warm dark matter is able to erase fluctuations on (present day) scales of < 1 Mpc, i.e. galaxy sized and below.

Cold dark matter comprises weakly interacting particles which become non-relativistic early and which therefore can diffuse only a negligible distance. The axion is a possible candidate for such a particle.

There are theoretical problems with scenarios for galaxy formation involving hot dark matter. Since small-scale fluctuations are erased, the formation process must occur through the fragmentation of supercluster-sized clouds into galaxies. However, studies of the dynamical collapse of superclusters indicate that they formed relatively recently at redshift z_{sc} in the range $0.5 < z_{sc} < 2$, while galaxies and quasars exist beyond $z = 3$. There are other problems with hot dark matter, including the difficulty of understanding how galaxies acquired massive halos. In particular, there is good observational evidence that dwarf galaxies with masses as low as 10^6–$10^8\ M_\odot$ have massive halos. It is hard to see how they can be retained, since initial fluctuations on this small scale would have been erased.

The cold dark matter scenario of galaxy formation suffers few problems of principle. Since the dark matter is decoupled from baryonic matter and from the radiation field which dominates the mass density, fluctuations in the dark matter can start to grow before the Universe becomes matter-dominated and before recombination. On the other hand, the baryonic matter is held by the smoothly distributed radiation until the era of decoupling. After recombination, the amplitude of baryon fluctuations grows rapidly to match those of the cold dark matter fluctuations which have had longer to grow. Thus, smaller mass fluctuations grow to non-linearity ($\Delta\rho/\rho > 1$), virialize and cluster within successively larger and larger bound systems. It can be shown that ordinary baryonic matter in gravitationally bound systems of mass 10^8–$10^{12}\ M_\odot$ cools within their dark matter halos to form galaxies while larger mass fluctuations form clusters of galaxies. The essential feature is that the dark matter forms potential wells into which the baryonic matter falls when it is released from the radiation field.

7.2. Numerical simulations

Simulations of the evolution of galaxy clustering in various model Universes dominated by cold dark matter have been calculated recently by Davis et al. (1985). They assumed a "constant curvature", $n = 1$, spectrum of initial fluctuations in the distribution of the dark matter and calculated models for $\Omega_0 = 1$ and $\Omega_0 < 1$, as well as one model with a finite, positive value of the cosmological constant Λ.

The distribution of baryonic matter was approximated by distributing 32768 particles in a box of size 32.5 $(\Omega_0 h^2)^{-1}$ Mpc, where $h = H_0/100$ is the present value of the Hubble constant. Two-dimensional projections of the particle distribution, i.e. the distribution of visible matter, of the resulting simulations at various phases in the expansion of the Universe were examined and compared with similar plots of the real galaxy distribution at the present epoch. In addition to making visual comparisons of the simulated galaxy distributions with the observed distributions, Davis et al. also calculated the two-point correlation function and random velocity distribution of the galaxies at various epochs in order to make quantitative comparisons with the observations.

The remaining results of the cold dark matter simulations were as follows:

(1) While there is a superficial resemblance with the observed galaxy distribution, models with $\Omega_0 = 1$ are inconsistent with observations if galaxies are assumed to be unbiased tracers of the underlying mass distribution. The random velocities of galaxies are predicted to be higher than is observed. In addition, it is not possible to simultaneously obtain the correct shape and amplitude of the galaxian two-point correlation function at the present epoch.

(2) For $\Omega_0 = 0.2$ the agreement with observations is better than with $\Omega_0 = 1$, but is still not adequate. A model with a positive, finite value of Λ resembles an open model with the same Ω.

(3) Accordingly, if galaxies sample the mass distribution no simulated cold dark matter simulation matches the observations.

(4) If galaxies only form at the peaks of the underlying mass distribution then it is possible to obtain consistency with the observations, both with $\Omega_0 = 0.2$ and $\Omega_0 = 1$. (The way in which this effects the simulations follows from the fact that the peaks in a random distribution are more highly correlated than random points. This may be seen by imagining a spectrum of white noise: the larger high-frequency fluctuations will tend to arise from places where the base provided by the low-frequency fluctuation is high.)

Thus the results of numerical simulations of the galaxy formation on the cold dark matter scenario forces us to the idea of biased galaxy formation in which galaxies form preferentially at places where the underlying density is high.

8. Conclusions

8.1. Summary

Largely from the results of large galaxy redshift surveys, we are steadily obtaining a clearer picture of the large-scale distribution of galaxies in space. These objects (and the quasars) seem to have formed at a redshift $z \sim 4$ (about 2 billion years after the Big Bang) when the scale of the Universe, measured by $1+z$, was not orders of magnitude different from its present scale. The process of galaxy formation was very effective; not much intergalactic gas remains. The cosmological density parameter measured at the present epoch has a value $\Omega_0 \sim 0.3$; the evidence on the baryonic component Ω_b derived from primordial nucleosynthesis shows that $\Omega_b < \Omega_0$ and that the mass density of the Universe is dominated by some form of dark matter. There is evidence that Ω_0 increases with the scale over which it is measured: it may be as high as $\Omega_0 = 0.5$ on scales of 50–100 Mpc. However, there is no observational evidence that $\Omega_0 = 1$, as the attractive "inflationary" Universe scenario demands.

The galaxies in the Universe around us at $z = 0$ are strongly clustered, with density fluctuations $\Delta\rho/\rho > 1$. We have no empirical information on how galaxy clustering has evolved, but the Lyman-alpha clouds (which may or may not be distributed like the galaxies) show no observable clustering in the redshift range $1.8 < z < 3.8$ over which they can be observed. Also, there is no detectable small-scale anisotropy in the microwave background radiation down to a level $\Delta T/T \sim 10^{-4}$ to 10^{-5}.

Although the existence of superclusters of galaxies points to an origin of galaxies in the form of "pancakes" and adiabatic fluctuations, the studies of absorption lines in quasars do not reveal gaseous pancakes before they have fragmented into galaxies. Moreover, the observed limits on microwave background fluctuations on scales of arcminutes are incompatible with adiabatic fluctuations unless the initial fluctuation spectra were very much steeper than that countenanced by theory.

Adiabatic fluctuations as the origin of large-scale structure in the Universe can in principle be saved by invoking "cold" dark matter as the dominant constituent of the Universe. However, detailed calculations show that this scenario only works if most of the mass of the Universe is non-baryonic and galaxies form preferentially in regions of high-matter density and not randomly.

Accordingly, the present view must be that not only is most of the matter in the Universe invisible, but that the visible galaxies are poor tracers of the mass distribution in the Universe.

8.2. *The future*

With larger optical telescopes, it would be possible to undertake deeper redshift surveys and directly observe the evolution of the galaxian correlation function at earlier times—say, out to a redshift of $z = 0.5$. The limits on the angular fluctuations in the microwave background must be pursued further on scales of arcminutes to degrees. It appears from current work that the limits on arcminutes scales could be pushed down to $\Delta T/T \sim 5 \times 10^{-6}$ by paying careful attention to sources of systematic errors. The clustering of the "heavy-element" absorption redshifts in quasar spectra could be used to investigate the distribution of galaxies at high redshifts and to settle the critical question of whether the Lyman-alpha clouds (which show no measurable clustering) are distributed in the same manner as the galaxies. Finally, on a more local scale, massive redshift surveys for galaxies out to a distance of 100 Mpc would enable us to evaluate Ω_0 on this scale.

8.3. Epilogue

When Niels Bohr was born, the science of cosmology as we know it today did not exist. The nature of the Universe of galaxies was demonstrated when Bohr was 40 and the expansion of the Universe was discovered when he was 44. At about this time, Lemaitre and Gamow began their bold speculations about the physics of the early Universe and identified many of the problems with concern us today—the problem of the formation of galaxies, the formation of the heavier elements and the creation of the relict radiation. It was the accidental discovery of this radiation by Penzias and Wilson (1965) three years after Bohr's death which unleashed the present activity in the study of the early Universe.

I believe that our present serious attempts to explore the evolution of the Universe back to at least the first few seconds represents one of the most remarkable developments in scientific history. Moreover, the possibility that the large-scale structure can ultimately be directly traced back to primordial quantum fluctuations would no doubt have pleased Bohr. And yet the most interesting question for Bohr's 200th birthday would surely be: "was the dark matter that cosmologists believed in during the late 20th century as ephemeral as the phlogiston of 200 years earlier?"

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Discussion, session chairman T.A. Bak

Bleuler: You assumed a perfectly new kind of matter (so-called dark matter) which (apart from gravitation) has no interaction whatever with visible matter. Is that not too much freedom undermining the basic assumption of understanding the universe through known physical laws?

Sargent: The existence of the dark matter has been inferred for more than 40 years and the evidence for it is becoming stronger. Apart from gravitation, the particles have to interact weakly with matter after the first second or so in the expansion of the Universe: there are several theoretical candidates for such particles.

Ginzburg: In the main part of your presentation you have assumed that the cosmological "constant" Λ is equal to zero. I am, however, convinced that there is at present no reason to assume that $\Lambda = 0$. What, in your argumentation and conclusions, depends on the hypothesis that $\Lambda = 0$?

Sargent: A finite Λ was introduced into some of the numerical simulations of the evolution of clustering in a Universe dominated by "cold" dark matter that I described. It did not help to resolve the problems that I outlined.

Jones: The only problem with introducing a cosmological constant is that we need to explain the remarkable coincidence that we are living at the time when $\Lambda \sim 3\ H_0^2$ (to within 90%). Appeal to anthropic principles is hardly a scientific explanation.